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Effect of variability and uncertainty on topological optimization of structures

WANG Ze	**Nawfal Blal** **David Dureisseix** **Béatrice Faverjon**

Résumé

Ce travail est conjoint à deux projets locaux MaDAMe_R2 et international (Australie - États-Unis) MURMUR à partir de LaMCoS. Le but de mon projet est principalement d'étudier l'effet de la variabilité et des incertitudes sur l'optimisation topologique des structures.

Dans ce domaine, il nous sera demandé de rechercher à l'échelle globale pour optimiser les structures macroscopiques sous différentes charges. L'analyse de la variabilité et des incertitudes concernera initialement le matériau, le chargement. L'effet de la topologie initiale sera pris en compte dans une seconde étape. L'étude sera basée sur une approche non intrusive couplant les résultats du modèle déterministe (sans variabilité) ainsi que des techniques de propagation des incertitudes.

Mots clé : *Optimisation de la Topologie, Matériau Isotrope Solide avec Pénalisation, Expansion du Chaos Polynomial, Simulation de Monte-Carlo*

Summary:

This work is joint to two local projects MaDAMe_R2 and international (Australia - United States) MURMUR starting at LaMCoS. The goal of my project is mainly to study the effect of variability and uncertainties on the topological optimization of structures.

In this subject, we will be requested to research on a global scale to optimize macroscopically structures under different loadings. The analysis of variability and uncertainties will initially concern the material, the loading. The effect of geometrical uncertainties of the material distribution will be studied in a second step. The study will be based on a non-intrusive approach coupling the results of the deterministic model (without variability) as well as propagation techniques of the uncertainties.

Key words: *Topology Optimization, Solid Isotropic Material with Penalization [1], Polynomial Chaos Expansion, Monte-Carlo Simulation*

Nomenclature

L	Length of the design domain
H	Width of the design domain
T	Thickness of the design domain
E_0	The Young's Modulus
ρ	The material distribution matrix
v	The Poisson's ratio
ρ_{min}	The minimum density of one element, in general 1e-3
ρ_{max}	The maximum density of one element, in general 1
F	The load
U	The displacement of a node
{f}	The unit vector of load
V	The volume real

K	The stiffness matrix
D	The stress matrix
λ	The Lagrange multiplier
q	The design parameter for SIMP
θ	The load orientation
ξ	The stochastic variable
ξ_c	The collocation points
e	The energy polynomial
e_i	The coefficient of the Legendre polynomial basis in energy polynomial
e_0	The first coefficient
ψ	Legendre polynomial basis
d	The given dimension of polynomial chaos expansion
p	The order of polynomial chaos expansion
N	The number of unknown polynomial coefficients
RSS	Residual sum of squares
n(i|p)	The number of samples of MC in the i-th energy distribution section
m(i|p)	The number of samples of PCE in the i-th energy distribution section
I	The number of energy distribution section in histogram

Content

1/ INTRODUCTION:

In recent years, the use of architectural materials has increased considerably in various industrial sectors (aeronautics, building, transport, nuclear, ...) thanks to (i) a better combination of several physical properties, (ii) improved performance of some existing materials. Topology optimization technology can greatly improve the efficiency of the use of materials.

Generally, structural topology optimizations are conducted in a deterministic manner, known as deterministic topology optimization (DTO) [1], where the design is determined without considering various sources of uncertainties. However, uncertainties are unavoidably observed in real-world applications due to insufficient knowledge, manufacturing errors, changeable environment, and so forth. This may lead to the vulnerable optimum structure or infeasible topologies due to the fluctuation of the structure performance. Therefore, there is a strongly increasing requirement to take the effect of uncertainty into consideration for optimal topologies in structural design.

In the case of uncertainty parameters defining probability distributions, the probabilistic description of the performance function is usually characterized by its statistical quantities such as mean value and standard deviation. In this scenario, a fundamental issue associated with robust topology optimization (RTO) [2] procedure is to calculate the statistical moments accurately and efficiently.

Probabilistic uncertainty propagation methods are often used in the analysis of a physical system to quantify the effects of uncertainties on estimated statistics of the system response. Such methods include the Monte-Carlo Simulation (MCS) method [5], the Stochastic Response Surface Method (SRSM) [3][4] and the Spectral Stochastic Finite Element Method (SSFEM) [5][6].

MCS are the most widely used sampling-based methods in robust design due to their accuracy and easy implementation but are time-consuming. Aside from the aforementioned MCS-method, SFEM require significant modification of existing deterministic numerical codes and become impossible for most engineers with no access to the source code of proprietary commercial software. SRSM, an alternative to the MCS method, using polynomial chaos is convergent in the mean-square sense, and replaces the numerical model with an approximated less-expensive surrogate model, which can be used to estimate the system response and analyze uncertainty propagation. Herein this article uses the SRSM.

The basic idea of SRSM is to approximate model inputs and outputs in terms of random variables such as standard normal variables by a Polynomial Chaos Expansion (PCE) [4][7][8]. For the solution of the PCE coefficients, the PCE method can be divided into intrusive and non-intrusive techniques. In the intrusive PCE method, the PCE coefficients are solved by the stochastic Galerkin projection [5], which requires access to the system equations and results in more complex system equations. Furthermore, if the mathematical model involves complex non-linearities, the Galerkin procedure can be a challenging task and difficult to implement. To overcome these difficulties, non-intrusive methods described below are useful.

In the non-intrusive PCE method, the system equations are treated as a black box and the calculation of PCE coefficients is based on a set of deterministic simulations, which is more amenable in terms of computational cost for large-scale models and in terms of modelling complexity for iterative methods. To calculate the PCE coefficients, two non-intrusive approaches can be used: the spectral projection method and the collocation-based method. The spectral projection method projects the output results into the base polynomials using an orthogonality property and multidimensional integral, which involves random sampling, quadrature, Strouds cubature formula, or sparse grid approaches.[5] The collocation-based method uses a linear regression algorithm that approximates the PCE coefficients to match the output results from the deterministic model at a set of collocation points using the least square algorithm, which is more straightforward to implement than the spectral projection.

This project studies the effect of the random variable of load orientation on a robust configuration design using the non-intrusive collocation-based PCE. Since the collocation-based method introduces additional approximations (least-square at the collocation points), its validation is performed with comparison to MCS in terms of accuracy and computational cost.

The report is organized as follows: A brief description of the deterministic topology optimization problem is presented in Section 2. In Section 3, the influence of parameter on the result is studied. The variability analysis of load orientation with MCS and geometry with CBSRSM are respectively illustrated in Section 4 and in Section 5. Conclusions are discussed in Section 6.

2/ TOPOLOGICAL OPTIMISATION PROBLEM

2.1/ Example under concern

This stochastic topology optimization example is studied on a simply supported beam. The design domain, the boundary, and loading conditions are illustrated in Figure 1. The dimensions of the beam are L = 90mm and H = 30mm and the thickness is T = 1mm. The isotropic material is assumed to have Young's modulus E_0 of 10MPa and Poisson's ratio v of 0.30. A single random load case is considered whose orientation is assumed to follow a uniform distribution with the interval of $[-3\pi/4, -\pi/4]$. The objective is to maximize the overall stiffness, which is equivalent to minimize the compliance. In robust design, the optimization problem consists in finding the optimal value of the random variable so that this regression equation has a minimum value there. The design domain is discretized by 2700 (90 × 30) four-node linear elements.

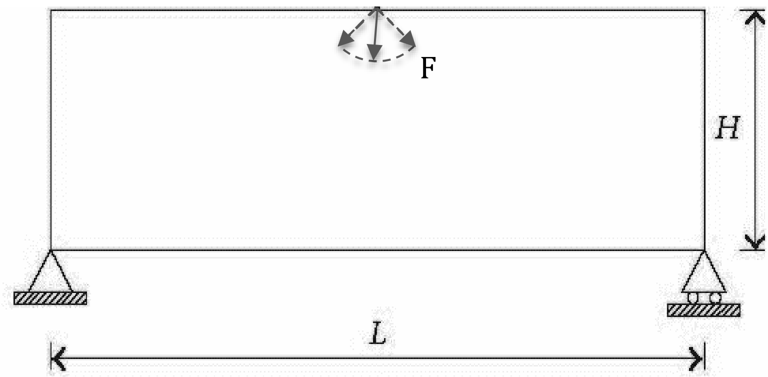


Figure 1 - Design domain, boundary conditions, and loading conditions of a 2D simply supported beam.

2.2/ Solid Isotropic Material with Penalization method [1]

Solid Isotropic Material with Penalization (SIMP) method uses the material distribution method and is based on the numerical calculation of the globally optimal distribution of the density of material ρ which is the design variable. For an interpolation scheme that properly penalizes intermediate densities (ρ in]0,1[) the resulting 0-1 (or black and white) design is actually the primary target of our scheme. The optimality criteria method for finding the optimal topology of a structure constructed from a single isotropic material then consists of the following steps:

Pre-processing of geometry and loading:

- Choose a suitable reference domain (the ground structure) that allows for the definition of surface tractions, fixed boundaries, etc.
- Choose the parts of the reference domain that should be designed, and other parts that should be left as solid domains or voids.
- Construct a finite element mesh for the ground structure, which should be fine enough in order to describe the structure in a reasonable resolution bit-map representation.
- Construct finite element spaces for the independent fields of displacements and the design variables.

Optimization:

Compute the optimal distribution over the reference domain of the design variable q. The optimization uses a displacement based finite element analysis and the optimality update criteria scheme for the density. The structure of the algorithm is:

- Make initial design, e.g., homogeneous distribution of material. The iterative part of the algorithm is then:
- For this distribution of density, compute by the finite element method the resulting displacements and strains.
- Compute the compliance of this design. If only marginal improvement (in compliance) over last design, stop the iterations. Else, continue. For detailed studies, stop when necessary conditions of optimality are satisfied.
- Update the density matrix, based on the equation (3). This step also consists of an inner iteration loop for finding the value of the Lagrange multiplier λ for the volume constraint.
- Repeat the iteration loop.

For a case where there are parts of the structure which are fixed (as solid and/or void) the updating of the design variables should only be invoked for the areas of the ground structure which are being redesigned (reinforced).

Post-processing of results:

- Interpret the optimal distribution of material as defining a shape, for example in the sense of a CAD representation.

Herein the SIMP program is considered as a black box, after the position of the concentrate load and boundary condition are defined. The inputs which can be modified are only four variables, load orientation, load amplitude, Young's modulus and Poisson's ratio.

In general, the topology optimization problem aims to obtain a design domain determined by the objective function.

$$\begin{cases} \min_{\rho} \quad f(\rho)=F^T U = F\{f\}^T U = F^2\{f\}^T K^{-1}\{f\} = \sum_{e=1}^{N_e} E_e(\rho) u_e^T k_e u_e \\ \quad s.t. \quad \mathrm{KU} = \mathrm{F}, \; V = \sum_{e\in N} \rho_e V_e \le V^*, \quad \rho_{min} < \rho_e < \rho_{max} \end{cases} \tag{1}$$

$$\begin{cases} \mathrm{K}(\rho) = \sum \mathrm{K_e}(\rho) = \sum \int \mathrm{B^T D_e}(\rho)\mathrm{B}\,\mathrm{d}\omega \\ \mathrm{D_e}(\rho) = \frac{\mathrm{E_e}(\rho)}{1-\mathrm{v}^2} \begin{bmatrix} 1 & \mathrm{v} & 0 \\ \mathrm{v} & 1 & 0 \\ 0 & 0 & \frac{1-\mathrm{v}}{2} \end{bmatrix} \\ E_e(\rho) = \rho_e^q E_o \end{cases} \tag{2}$$

$$\rho_e(\mathrm{k+1}) = \rho_e(k) \cdot \lambda^{-1} q \rho_e^{q-1} E_o u_e^T k_e u_e \text{ (OCUpdate)} \tag{3}$$

2.3/ Numerical results

The deterministic topological optimization results caused by different inputs are shown in Figure 2 and Figure 3. The Figure 2 shows these configurations caused by different Poisson's ratio v∈(0.0 , 0.5) when the Young's Modulus is E=10MPa, the load amplitude F=10N, the load orientation θ=π/2. With the same the Young's Modulus E=10MPa, load amplitude F=10N, and a Poisson's ratio v=π/2, different load orientation θ∈[π/4 , 3π/4] leads to the configurations in the Figure 3.

3/ Parametric studies

3.1/ Influence of Young's modulus and load amplitude

According to the performance function, we found that the Young's modulus and the load amplitude have a linear relationship with the compliance, so we consider that they would not have any effect on the optimization result, which has been proved by experiments.

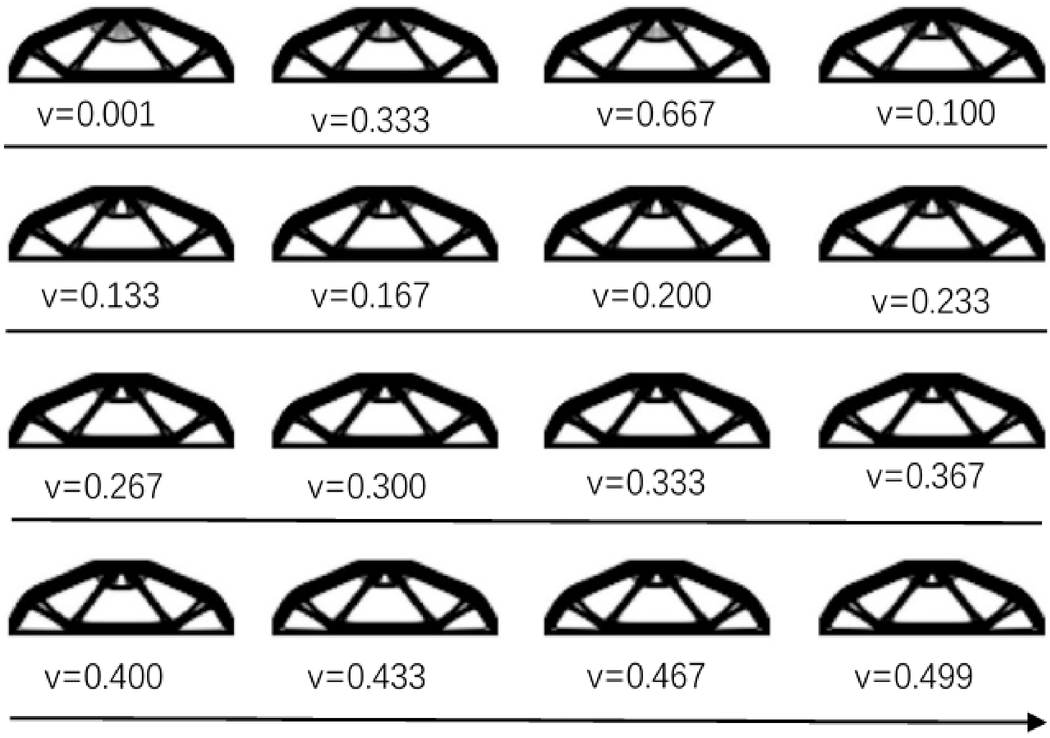


Figure 2 – Influence of Poisson's ratio

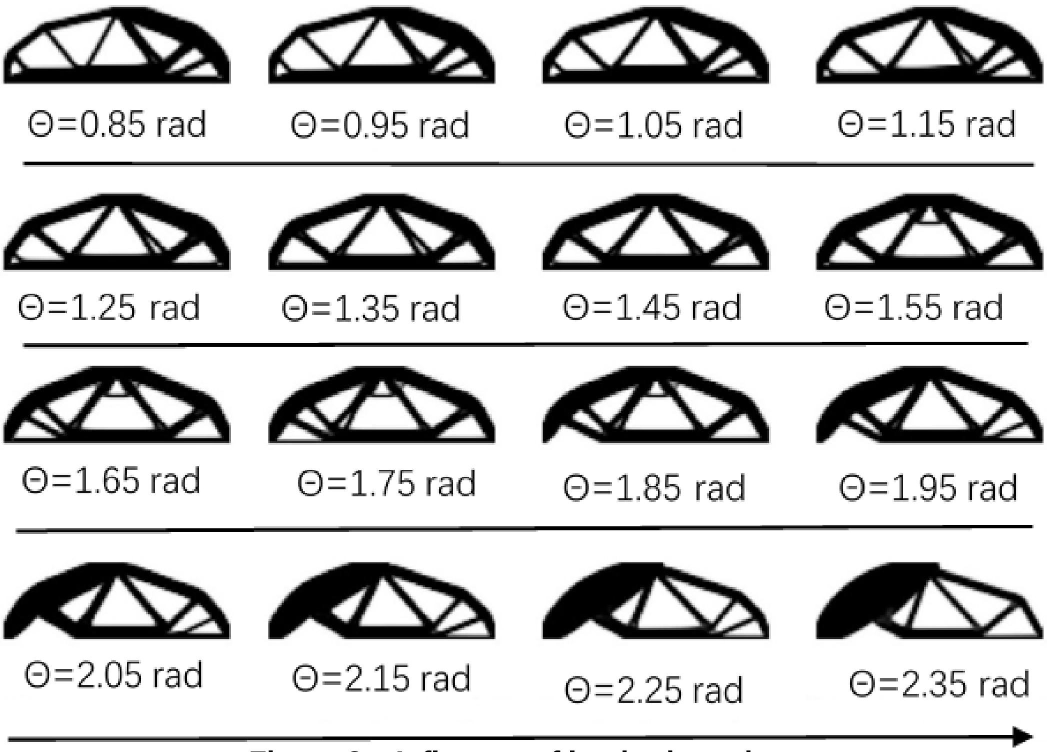


Figure 3 – Influence of load orientation

3.2/ Influence of Poisson's ratio

Figure 2 indicates that the Poisson's ratio has almost no effect on topological optimization of structures.

Although the stress matrix is a non-linear function of the Poisson's ratio as shown in equation (2), in the OCUpdate process, the new density is calculated using the same stress matrix for each element. Therefore, the results of topology optimization have not changed substantially. However, in detail, Poisson's ratio can be considered as a filter because a larger Poisson's ratio can avoid the smaller structures such as small holes or small branches, so that the outline of the structure becomes clearer

3.3/ Influence of load orientation

Figure 3 shows that the load orientation has a great influence on the topology optimization of the structure.

As the equation (1) shows, the performance function is the product of the applied load and the displacement in the direction of the load, which is also the sum of the deformation energy of each element. Therefore, different load orientations result in different unit load vectors, and unit vectors in different directions let each element have a different deformation. Subsequently, in the OCUpdate process (equation 3), the new density matrix will change greatly, which means that the material distribution of the new structure has changed greatly compared to before.

4/ Variability analysis of load orientation

4.1/ Monte Carlo Simulations

The Monte Carlo simulations (MCS) are the most widely used sampling-based methods in robust design due to their accuracy and easy implementation. Using MCS method herein, we sample randomly and uniformly 10,000 values in the load orientation distribution range ($[-\frac{3\pi}{4}, -\frac{\pi}{4}]$) as the inputs of the determinist topological optimization problems. Solving them, we record each density matrix.

Once a new density matrix is recorded, we compute the mean of it and all previous density matrices and then calculate respectively the deformation energies produced by applying these loads on the mean structure as well as the mean and standard deviation of these deformation energies. At last, we draw respectively the function curves of the error in mean and standard deviation of energy (using 10,000 realizations as the reference) along with the number of realizations in Figures 4 and 5. The reference result e^{∞} corresponds to the 10000 simulations result.

4.2/ Numerical results

The Figures 4 and 5 indicate that MCS method has an excellent accuracy and both curves weaken the shock when the number of realizations reaches 1,000 and converge respectively to 2.03e-5 (mean) and 5.8e-6 (std) when the number of realizations reaches 4,000. It means that under the setting condition we can get the robust mean structure with 4000 realizations, which is shown in Figure 6. Thus we can conclude that under such a random uniform distributed load, the expected deformation energy of the mean structure is 2.03×10^{-5}, with a standard variance of 5.81×10^{-6}. Unfortunately, this structure is not the most ideal result because there is a large fuzzy area with its density between 0 and 1 in the configuration, which is not acceptable in the manufacturing process.

5/ Variability analysis of the geometry

5.1/ Random inputs of the problem

Normally, in the manufacturing process, it is inevitable that the material contains impurities, which will affect its performance. Usually this effect is a random distribution, so we use stochastic input to characterize this effect. The goal of the analysis of geometric

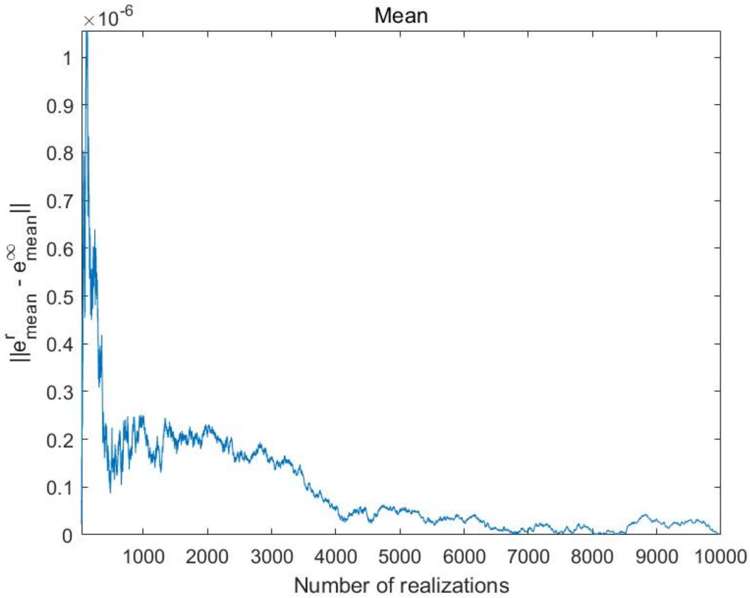


Figure 4 – The mean of deformation energies converges to 2.03e-5

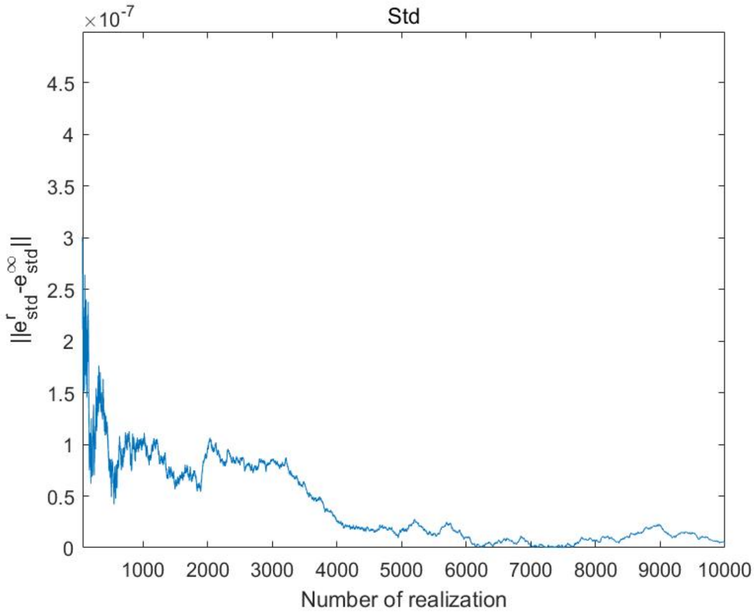


Figure 5 – The standard deviation of deformation energies converges to 5.8e-6

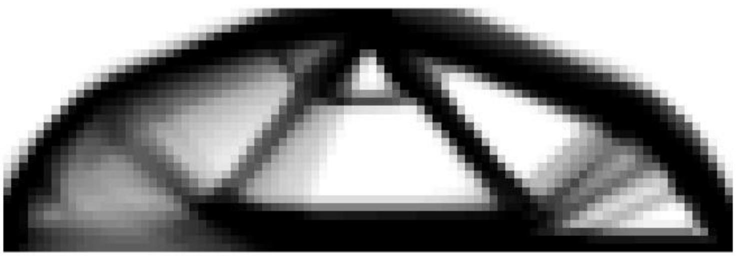


Figure 6 – Mean structure

variability is to study the effect of this random distribution on the strain energy of the structure under load.

The first step in the implementation of the SRSM is to represent all the stochastic inputs in terms of standard random variables (SRVs)[4]. Here we consider that the density of the structure obeys the uniform distribution and we thus have two models, the first:

$$\rho(\xi, M) = \xi\bar{\rho}(M) \tag{4}$$

where ξ is the variable which is uniformly distributed and takes a value within [0.9,1], and $\bar{\rho}(t)$ is the density matrix of the optimization result for a given θ.
The second model:

$$\rho(\xi, M) = \bar{\rho}(M) + \xi \tag{5}$$

where ξ is the variable which is uniformly distributed and takes a value within [-0.1,0.1], and $\bar{\rho}(t)$ is the same as above.

5.2/ Probabilistic collocation method

The output of an analysis model is clearly influenced by all of the inputs. Therefore, any general functional representation of uncertainty in model outputs should take into account uncertainties in all inputs. The stochastic method used here is the stochastic response surface method using collocation-based PCE (CBPCE). The uncertain output is explained by:

$$e(\xi) = \sum_{j=0}^{N} e_j \psi_j(\xi) \tag{6}$$

where the number of unknown polynomial coefficients is equal to $N = \sum_{s=1}^{p} \frac{1}{s!} \prod_{r=0}^{s-1} (d + r) = \frac{(d+p)!}{d!\,p!} - 1$ with p the order of polynomial chaos and d the given dimension of PCE or the number of random variables of inputs $\xi = (\xi_1, \xi_2 \ldots \xi_d)$. The random inputs follow the uniform probability law and the base polynomials ψ_j are mutually orthogonal. We use Legendre polynomials [9], which can be obtained as:

$$\begin{aligned} \psi_{(0)} &= 1 \\ \psi_{(1)} &= x \\ \psi_{(n)} &= \frac{2n-1}{n} x\psi_{(n-1)}(x) - \frac{n-1}{n}\psi_{(n-2)}(x) \end{aligned} \tag{7}$$

The first six Legendre polynomials are shown as Figure 7.

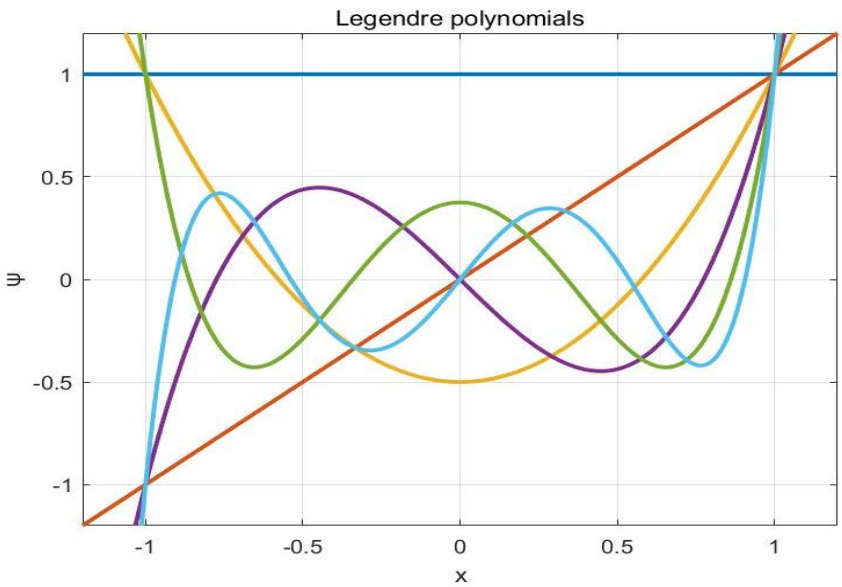


Figure 7 – the first five Legendre polynomials

The collocation-based method outputs are calculated at a set of collocation points $\xi_c = (\xi_1, \xi_2 \ldots \xi_l)$ in the parameter space from the deterministic model. The following two criteria should be satisfied in the selection of collocation points. The first criterion is that the selected collocation points must be chosen as the roots of at least higher 1-order polynomial to capture the points from the region of high probability. The second criterion is that the collocation points should be selected such that the overall distribution of the collocation points is more symmetric with respect to the origin. If still more points are available, the collocation points are selected randomly.

For this reason that the probabilistic collocation method is inherently unstable, a regression analysis is proposed arising from the minimization of the least squares norm of the residual:

$$\min_{\xi_c} \left\| e(\xi_c) - \sum_{i=0}^{N} e_i \psi_i(\xi_c) \right\|^2 \quad (8)$$

It is often used to estimate the unknown coefficients in the polynomial chaos expansion. In the regression-based collocation method, the number of collocation points is selected in the same way as discussed previously but must be higher than the number of unknown coefficients to be estimated. In order to obtain robust estimates of the unknown coefficients, it should better be twice the number of unknown coefficients.

5.3/ The first model: Numerical results and Comparisons with Monte Carlo method

According to the PCE method, for finding the number of unknown polynomial coefficients N, the order of polynomial chaos p and the dimension of PCE M should be gotten firstly. Herein d is known as 1 due to the only one stochastic variable, but the order of polynomial chaos p is unknown. Therefore in order to find a suitable p-value, we should use the results of the MC method as a reference to judge the outcome of the PCE method for each feasible p-value.

From Figure 8-(a1) obtained by using the method in Section 4, it is obvious that the energy mean converges to 2.2846e-6, when the number of random variable samples is greater than 10,000. In addition, according to the equation (6), the energy function can be also expressed as

$$\mathrm{e}(\xi) = e_0 + \sum_{j=1}^{N} e_j \psi_j(\xi) \quad (9)$$

where e_0 happens to be the energy mean. Consequently, we obtain the energy mean for the first 15 p-values by regression analysis and get their error relative to the above energy mean 2.2846e-6. As shown in Figure 8-(b1), no matter how the p-value changes, the energy mean obtained by PCE method is unaffected and close to 2.2846e-6. Subsequently, shown in Figure 8-(c1), we obtain the residual sum of squares (RSS) of the probability distribution between PCE method and MC method by counting the number of samples in each energy distribution section, with the equation:

$$\mathrm{RSS(p)} = \sum_{i=1}^{l} \big(n(i|p) - m(i|p)\big)^2 \quad (10)$$

where l is the number of energy distribution section in histogram, n(i) and m(i) are respectively the number of samples of MC and PCE in the i-th energy distribution section. Combining these two figures, it is easy to find that the minimum p-value is 2, and the N-value is 2.

According to the first criterion in the selection of collocation points, we should choose the roots of the 3-order Legendre polynomial as the value of the collocation points. Thus, the regression analysis (equation-8) is written as:

$$\begin{bmatrix} 1 & x_1 & 1.5x_1^2 - 0.5 \\ 1 & x_2 & 1.5x_2^2 - 0.5 \\ 1 & x_3 & 1.5x_3^2 - 0.5 \end{bmatrix}_{3\times 3} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} e(x_1) \\ e(x_2) \\ e(x_3) \end{pmatrix} \quad (11)$$

Where x_1, x_2, x_3 are the roots, which value respectively $\pm 0.7746, 0$. $e(x_1), e(x_2), e(x_3)$ are structure deformation energies solved from the FEM Black-Box.

The energy PCE is concluded as $e(x) = 2.2829 \times 10^{-6} - 1.2035 \times 10^{-7}x + 6.3344 \times 10^{-9}x^2 + \mathrm{O}(x^3)$, which is drawn in Figure 8-(d1). As the result, the probability distribution function is displayed in Figure 8-(e1), where we conclude that the PCE method and the MC method provide the same accurate results. However, I must mention here that the MC method takes nearly 40 minutes but the PCE method takes merely less than one second, which reflect the efficiency of the PCE method.

Moreover, according to the first criterion in selection collocation points, the collocation points must be chosen as the roots of at least higher 1-order polynomial, and in general the number of collocation points greater than the number of unknown coefficients, the PCE is more robust. So we utilize the 8 collocation points arising from the roots of 8-order Legendre polynomial to fit a quadratic PCE which is $e(x) = 2.2829 \times 10^{-6} - 1.2039 \times 10^{-7}x + 6.3403 \times 10^{-8}x^2 + \mathrm{O}(x^3)$. This equation has almost no difference with the former. Therefore, we think the 4-collocation-points PEC has enough accuracy to approximate the nonlinear process, which also reflects that a second-order PCE is sufficient to describe the first model and the p-value of 2 is optimal.

5.4/ The second model: Numerical results and Comparisons with Monte Carlo method

The second model is different from the first model, because adding or subtracting directly a stochastic variable to the element density will make the density value out of the permitted range [0,1]. Therefore we make a regulation that the density of each element out of the permitted range is reset as the boundary value 0 or 1. Subsequently the same analysis process is implemented as shown in Figure 8-(a2), and the energy mean obtained from 10,000 randomly selected samples is used as a reference. Figure 8-(b2) and Figure 8-(c2) are obtained by the above method, and the energy PCE could be written as $e(x) = 2.17 \times 10^{-6} - 2.73 \times 10^{-7}x + 3.39 \times 10^{-8}x^2 + 9.19 \times 10^{-8}x^3 + 5.78 \times 10^{-8}x^4 - 1.10 \times 10^{-7}x^5 - 8.77 \times 10^{-8}x^6 + 4.85 \times 10^{-8}x^7 + 4.13 \times 10^{-8}x^8 + O(x^8)$ which is drawn in Figure 8-(d2). The probability distribution function is displayed in Figure 8-(e2).

Observing Figure 8-(b2) and Figure 8-(c2), we know that once the order is greater than 8, the PCE method will have the same accuracy as the MC method, but we don't know if this is due to the increase in the number of collocation points or the increase in the number of polynomial orders. Thus, as a comparative experimental, we specify that there is always 20 collocation points for each p-value analysis, and the same analysis process is shown in Figure 9. We can conclude that the more collocation points as used, the more accurate is the energy mean e_0 in Equation (9) but the number of collocation points have no effect on the probability distribution function due to the reason that Figure 9-(b) has no difference from Figure 8-(c2). Consequently the 8-orders PCE is optimal to describe the second model.

Compared to the Model 1, Model 2 has more compatibility with the physical environment and can produce accurate results. However, the order of his equation is too high, and the expression of the equation is extremely unstable because the coefficient of the high-order term is large and mutable.

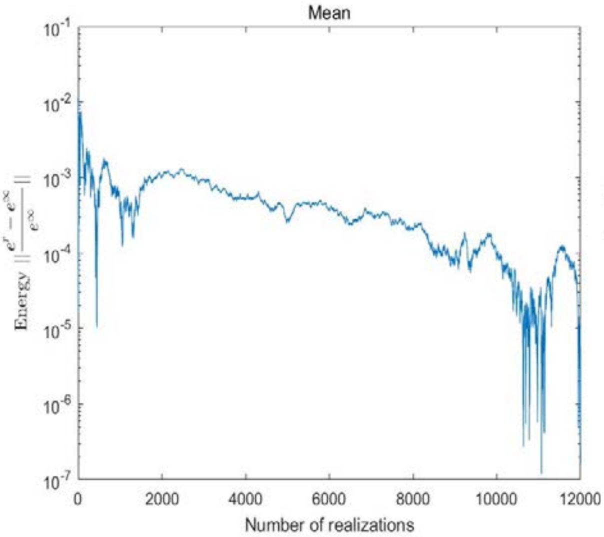
6/ Conclusion

Uncertainties are unavoidably observed in real-world applications due to insufficient knowledge, manufacturing errors, changeable environment, and so forth. This may lead to the vulnerable optimum structure or infeasible topologies due to the fluctuation of the structure performance. Therefore, there is a strongly increasing requirement to take the effect of uncertainty into consideration for optimal topologies in structural design. The report aims to study the effect of variability and uncertainty on topological optimization of structures, which consists mainly of three parts: one is parametric studies, one is variability analysis of load orientation, and another is variability analysis of the geometry.

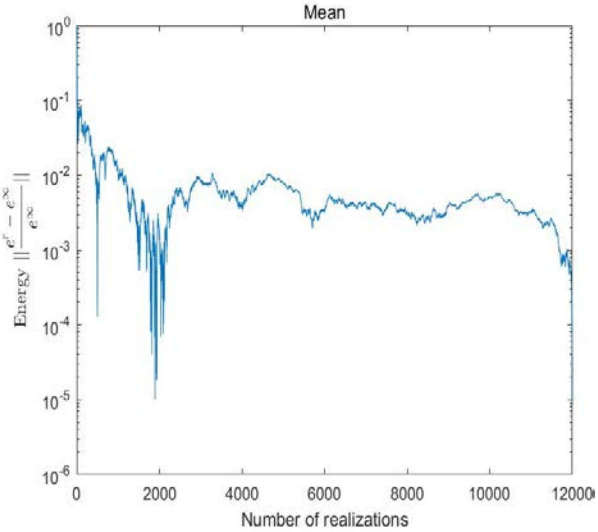
In the first section, we have got some important conclusions that i) Young's modulus and load amplitude have not any effect on the topological optimization of structures, ii) the Poisson's ratio has almost no effect on the optimization result, and iii) the load orientation has a great influence.

In the second section, we used the Monte Carlo method to make the variability analysis of the geometry. Sampling randomly 10,000 load directions, we got the mean structure by solving the average of these 10,000 independent topological optimization results, which has a convergent mean and standard deviation of energy produced by applying uniformly and stochastically a load at it.

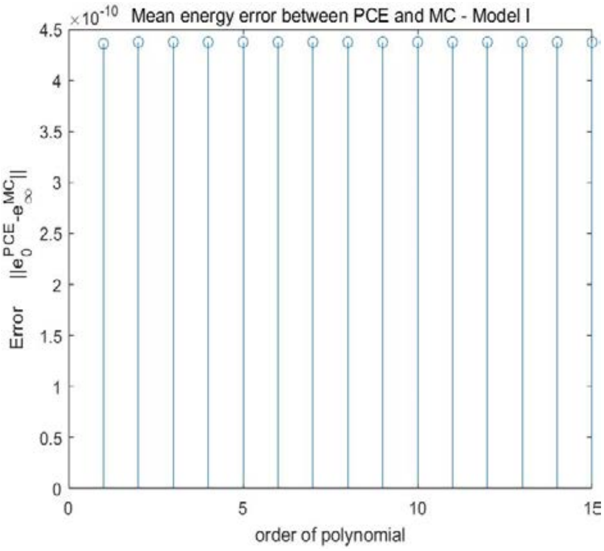
In the third section, for a given configuration, the effect of changes in material density on the structural deformation energy is studied. This analysis presented the application of SRSM to the propagation of parameter uncertainty in the model. With compared to the MC method, CBSRSM has same accuracy and lower cost.



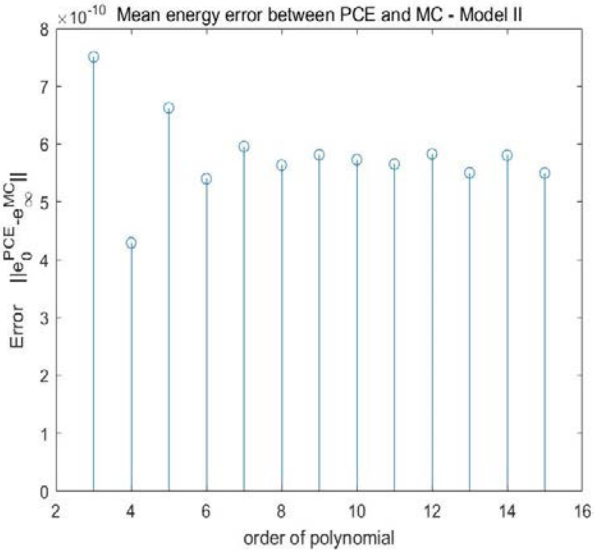
(a-1)



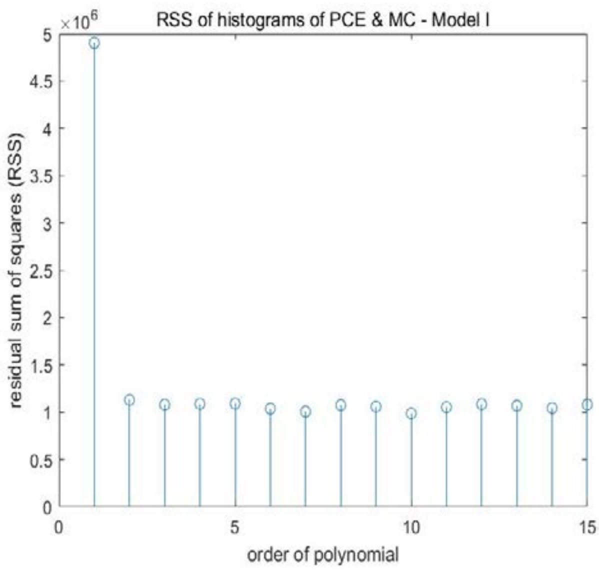
(a-2)



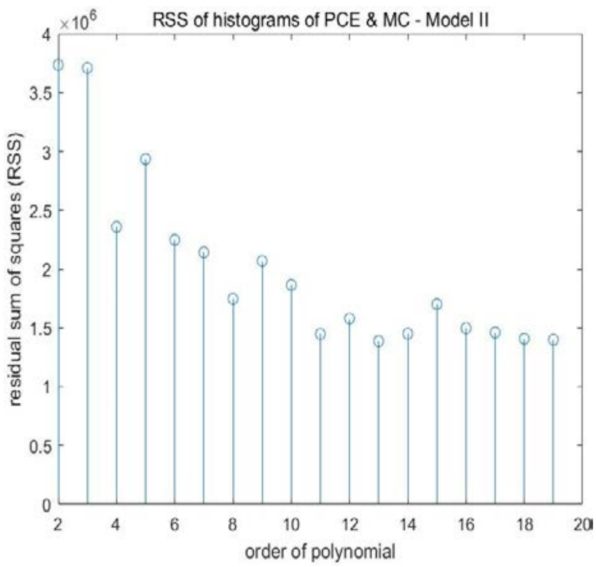
(b-1)



(b-2)



(c-1)



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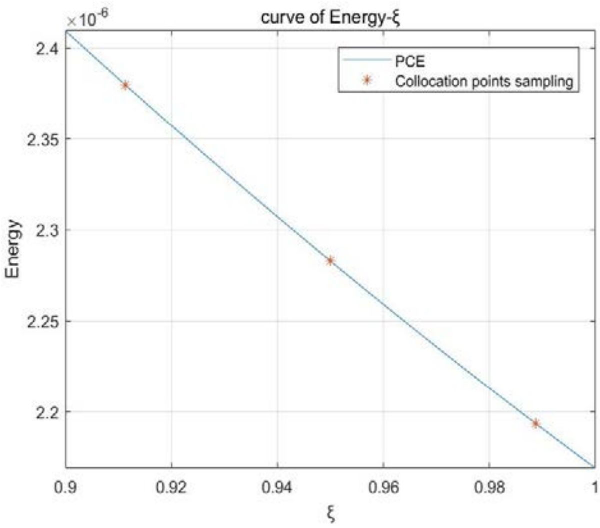
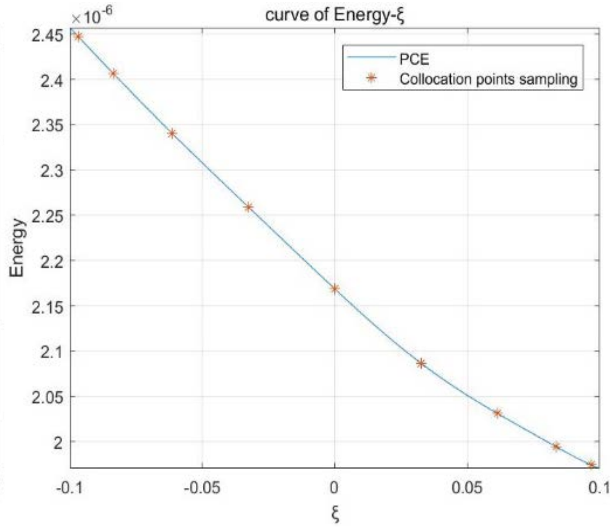


(d-1)



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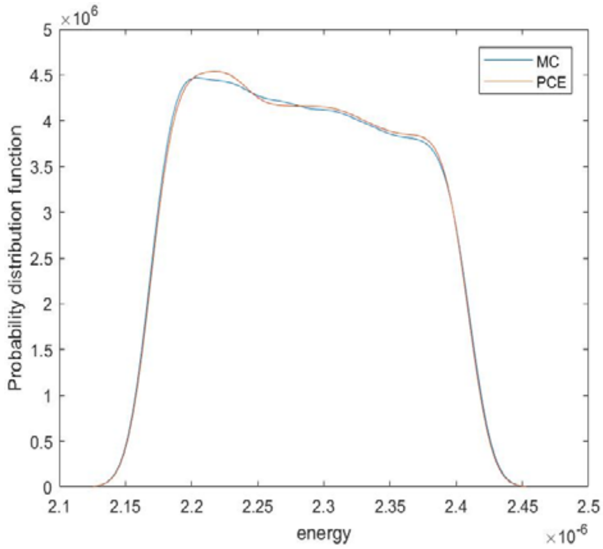
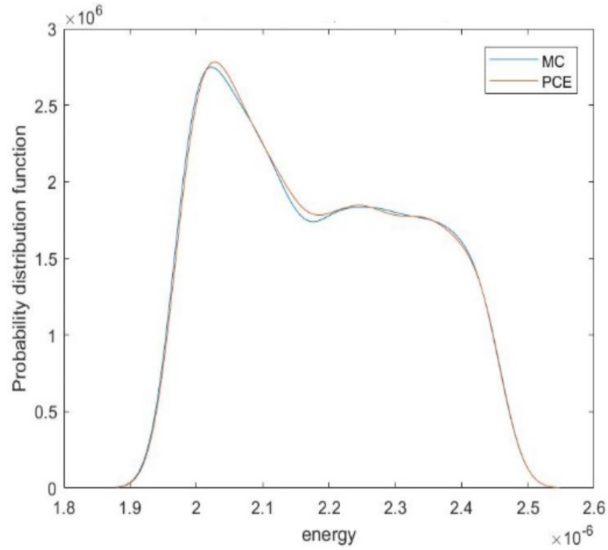
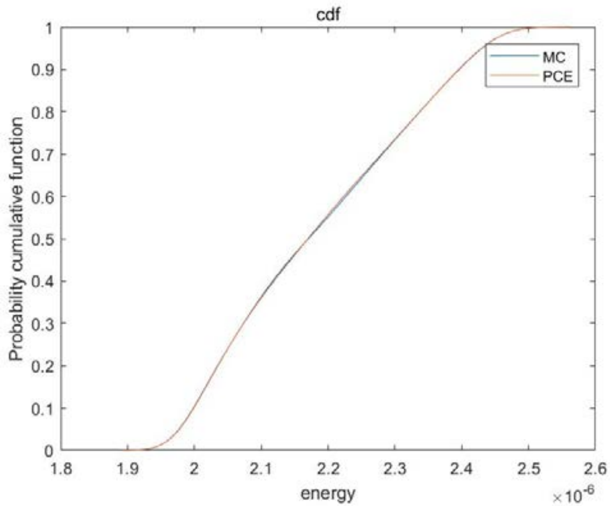


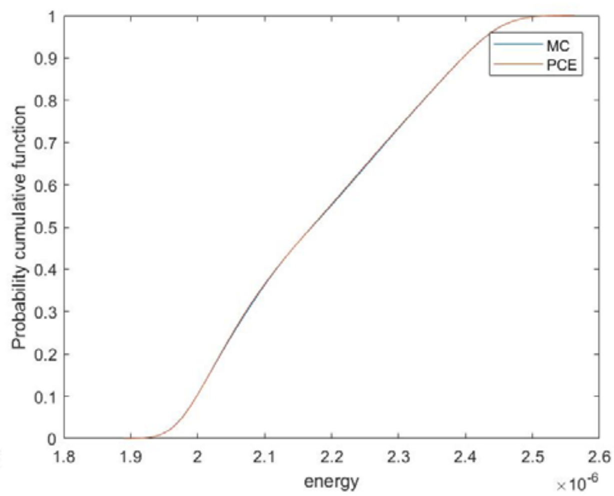
(e-1)



(e-1)

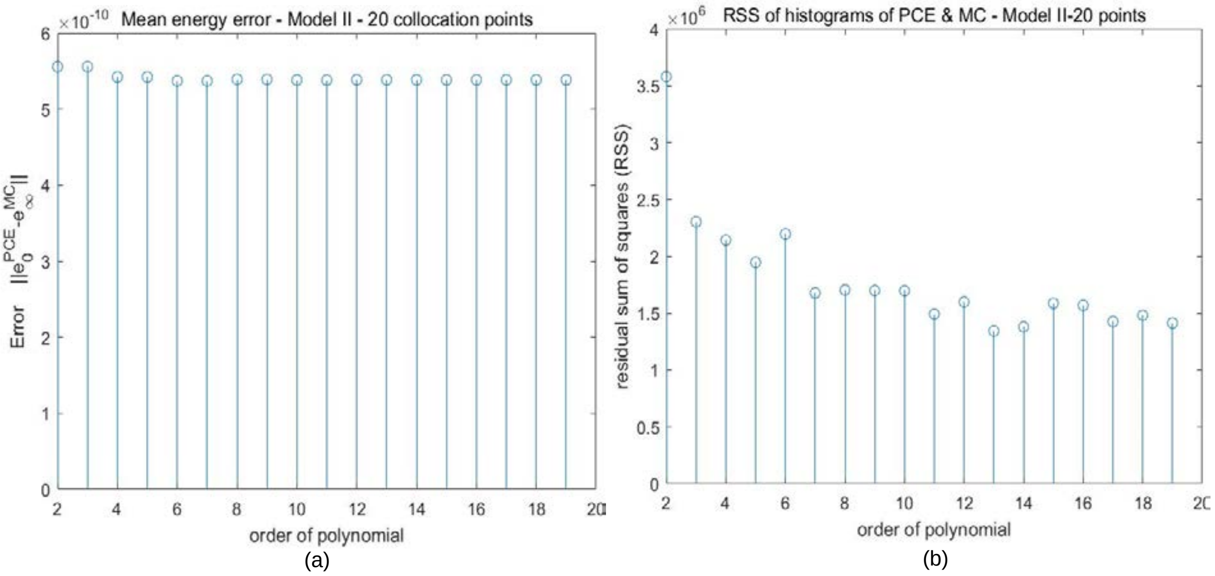


(f-1)



(f-2)

Figure 8 – (a) Evaluation of the number of realizations
(b) Mean energy error between PCE and MC with 20 collocation points
(c) Residual sum of squares of histograms of PCE&MC with 20 collocation points
(d) Curve of Polynomial Chaos Expansion
(e) Probability distribution function
(f) Probability cumulative function



(a) (b)

Figure 9 – (a) Mean energy error between PCE and MC with 20 collocation points
(b) Residual sum of squares of histograms of PCE&MC with 20 collocation points

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